6. Trigonometry

1 mark Questions

1. The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ is equal to

   (A) $\tan^2\theta$ **(B) 1** (C) $\cot^2\theta$ (D) 0

2. $\tan\theta \operatorname{cosec}^2\theta - \tan\theta$ is equal to PTA-3

   (A) $\sec\theta$ (B) $\cot^2\theta$ (C) $\sin\theta$ **(D) $\cot\theta$**

3. If $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$, then the value of k is equal to PTA-1

   (A) 9 **(B) 7** (C) 5 (D) 3

4. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \operatorname{cosec}\theta = b$, then the value of $b(a^2 - 1)$ is equal to

   (A) $2a$ (B) $3a$ (C) 0 (D) $2ab$

5. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, then $x^2 - \frac{1}{x^2}$ is equal to PTA-2

   (A) 25 **(B) $\frac{1}{25}$** (C) 5 (D) 1

6. If $\sin\theta = \cos\theta$, then $2\tan^2\theta + \sin^2\theta - 1$ is equal to PTA-1, 4

   (A) $\frac{-3}{2}$ **(B) $\frac{3}{2}$** (C) $\frac{2}{3}$ (D) $\frac{-2}{3}$

7. If $x = a\tan\theta$ and $y = b\sec\theta$ then

   (A) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

8. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$ is equal to

   (A) 0 (B) 1 **(C) 2** (D) -1

9. $a\cot\theta + b\operatorname{cosec}\theta = p$ and $b\cot\theta + a\operatorname{cosec}\theta = q$ then $p^2 - q^2$ is equal to PTA-5

   (A) $a^2 - b^2$ **(B) $b^2 - a^2$** (C) $a^2 + b^2$ (D) $b - a$

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$, then the angle of elevation of the sun has measure PTA-6, SEP-21

    (A) 45° (B) 30° (C) 90° **(D) 60°**

11. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point 'b' metres above the first, the depression of the foot of the pole is 60°. The height of the pole (in metres) is equal to

    (A) $\sqrt{3}\,b$ **(B) $\frac{b}{3}$** (C) $\frac{b}{2}$ (D) $\frac{b}{\sqrt{3}}$

12. A tower is 60 m height. Its shadow is x metres shorter when the sun's altitude is 45° than when it has been 30°, then x is equal to MAY-22

    (A) 41.92 m **(B) 43.92 m** (C) 43 m (D) 45.6 m

13. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in meters) is

    (A) $20, 10\sqrt{3}$ (B) $30, 5\sqrt{3}$ (C) 20, 10 **(D) $30, 10\sqrt{3}$**

14. Two persons are standing 'x' metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is

(A) $\sqrt{2}x$ **(B)** $\frac{x}{2\sqrt{2}}$ (C) $\frac{x}{\sqrt{2}}$ (D) $2x$

15. The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is JUL-22

(A) $\frac{h(1+\tan\beta)}{1-\tan\beta}$ (B) $\frac{h(1-\tan\beta)}{1+\tan\beta}$ (C) $h\tan(45^0-\beta)$ (D) none of these

2 mark Questions

1. Prove the following identities (i) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$ SEP-20

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}$$

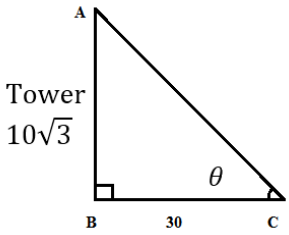
$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

2. Find the angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower of height $10\sqrt{3}m$. PTA-2, SEP-21, JUL-22



In right angle ΔABC

AB = Tower

$= 10\sqrt{3}m$

$BC = 30m$

$tan\theta = \frac{AB}{BC}$

$= \frac{10\sqrt{3}}{30} = \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

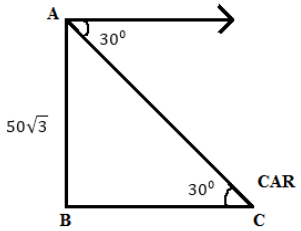
$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30^\circ$

$\therefore$ The angle of elevation $\boldsymbol{\theta = 30^\circ}$

3 . From the top of a rock $50\sqrt{3}m$ high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock. PTA-6, MAY-22



AB = Height of the rock $= 50\sqrt{3}$

Angle of depression $= 30^\circ$

In right angle $\Delta\ ABC$,

$$\tan 30^\circ = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$$

$$\frac{50\sqrt{3}}{BC} = \frac{1}{\sqrt{3}}$$

$$BC = 50\sqrt{3} \times \sqrt{3} = 50 \times 3$$

$$= 150m$$

The distance of the car from rock

$$= \mathbf{150\ m}$$

5 mark Questions

1. If $\sqrt{3}\sin\theta - \cos\theta = 0$, then show that $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$ PTA-3

Given: $\sqrt{3}\sin\theta - \cos\theta = 0$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^0 \qquad \left(\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$$

LHS: $\tan 3\theta = \tan 3(30^\circ) = \tan 90^\circ = \infty$ ..... (1)

RHS:

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \frac{3\times\tan 30^\circ - \tan^3 30^\circ}{1 - 3\tan^2 30^\circ}$$

$$= \frac{3\times\frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3}{1 - 3\times\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{3}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3}{1 - 3\times\frac{1}{3}}$$

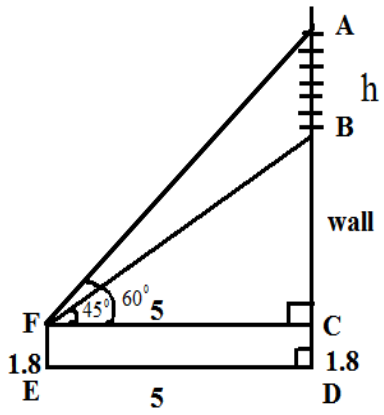
$$= \frac{\sqrt{3} - \left(\frac{1}{\sqrt{3}}\right)^3}{0} = \infty$$ ..... (2)

(1) = (2)

$$\therefore \mathbf{\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}}$$

2. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5m away from the wall, what is the height of the window? $(\sqrt{3} = 1.732)$.

JUL-22



Let AB = Window = h

EF = Man = $180cm$

= $1.8m = CD$

$CF = 5m$

To find the height of the window

In right angle $\Delta\, BCF$

$$\tan 45^\circ = \frac{BC}{5}$$

$$1 = \frac{BC}{5}$$

$\therefore BC = 5\text{m}$

In right angle ΔACF

$$\tan 60^\circ = \frac{AC}{5}$$

$$\sqrt{3} = \frac{AC}{5}$$

$$AC = 5\sqrt{3}$$

$$BC + AB = 5\sqrt{3}$$

$$5 + h = 5\sqrt{3}$$

$$h = 5\sqrt{3} - 5$$

$$= (5 \times 1.732) - 5$$

$$= 8.660 - 5$$

$$h = 3.66\text{m}$$

Height of the window $\boldsymbol{h = 3.66\text{m}}$

3. From the top of the tower 60 m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post. $(\tan 38^\circ = 0.7813, \sqrt{3} = 1.732)$

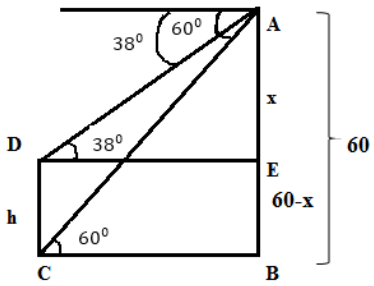
SEP-20

AB =Tower $= 60m$

CD =lamp post $= h$

$AE = x$

$CD = BE = 60 - x = h$



In right angle $\Delta\ AEC$

$\tan 38^\circ = \frac{AE}{DE} = 0.7813$

$DE = \frac{x}{0.7813}$..... (1)

In right angle $\Delta\ ABC$

$\theta = 60^\circ$

$\tan 60^\circ = \frac{AB}{BC} = \sqrt{3}$

$\frac{60}{BC} = \sqrt{3}$

$BC = \frac{60}{\sqrt{3}}$

$BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{60\sqrt{3}}{3}$

$BC = 20\sqrt{3}$

$BC = DE$

$\therefore DE = 20\sqrt{3}$ ..... (2)

From (1) & (2)

$DE \Rightarrow \frac{x}{0.7813} = 20\sqrt{3}$

$x = 20\sqrt{3} \times 0.7813$

$x = 20 \times 1.732 \times 0.7813$

$x = 27.064m$

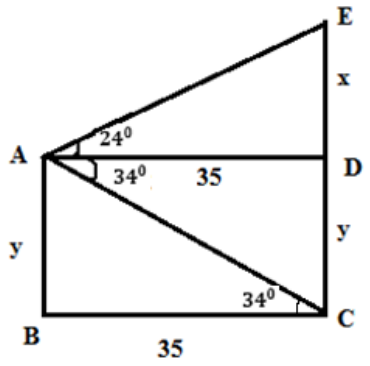
Height of the lamp post

$h = 60 - x$

$= 60 - 27.064$

$\boldsymbol{h = 32.93m}$

4. A building and a statue are in opposite side of a street from each other 35 m apart. From a point on the roof of building the angle of elevation of the top of statue is 24° and the angle of depression of top of the statue is 34°. Find the height of the statue. $(\tan 24^{\circ} = 0.4452, \tan 34^{\circ} = 0.6745)$ PTA-4



AB = Building = y

CE = State = $x + y$

$BC = AD = 35m$

In right angle $\Delta\, ADE$

$\tan 24^0 = \frac{ED}{AD} = 0.4452$

$\frac{x}{35} = 0.4452$

$x = 35 \times 0.4452$

$x = 15.582$

In right angle ΔABC,

$\tan 34^{\circ} = \frac{AB}{BC} = 0.6745$

$\frac{y}{35} = 0.6745$

$y = 0.6745 \times 35$

$= 23.6075$

Height of the statues

$CE = x + y$

$= 15.582 + 23.608$

$= 39.190$

$CE = 39.19m$

Height of the statue = $\mathbf{39.19m}$